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MATHEMATICAL MODELING OF INFILTRATION OF GROUND WATER INTO SURROUNDING LARGE BASIN AREA

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Abstract

Infiltration of ground water on the ground water surface has been modeled in the form of non-linear Boussinesq's equation. The height of water mound is obtained as a function of time t and distance x using group theoretical approach. Its graphical representation has been obtained by using Mat lab coding.

1. Introduction

Ground water is extremely important to our way of life. Most drinking water supplies and often irrigation water for agricultural needs are drawn from underground sources. More than 90% of the liquid fresh water available on or near earth's surface is groundwater. Groundwater is derived from rain and melting snow that percolate downward from the surface; and collected in the open pore spaces between soil particles or in cracks and fissures in bedrock. The process of percolation is called infiltration.

Infiltration is the process by which water on the ground surface enters the soil. Infiltration rate in soil science is a measure of the rate at which soil is able to absorb rainfall or irrigation. It is measured in inches per hour or millimeters per hour. The rate

Key Words and Phrases : *Infiltration, Lie-group analysis, Non-linear Boussinesq's equation.*

decreases as the soil becomes saturated. If the precipitation rate exceeds the infiltration rate, runoff will usually occur unless there is some physical barrier. It is related to the saturated hydraulic conductivity of the near-surface soil.

Infiltration is governed by two forces: gravity and capillary action. While smaller pores offer greater resistance to gravity, very small pores pull water through capillary action in addition to and even against the force of gravity. The rate of infiltration is affected by soil characteristics including ease of entry, storage capacity, and transmission rate through the soil. The soil texture and structure, vegetation types and cover, water content of the soil, soil temperature, and rainfall intensity all play a role in controlling infiltration rate and capacity. For example, coarse-grained sandy soils have large spaces between each grain and allow water to infiltrate quickly.Vegetation creates more porous soils by both protecting the soil from pounding rainfall, which can close natural gaps between soil particles, and loosening soil through root action. This is why forested areas have the highest infiltration rates of any vegetative types.

The top layer of leaf litter that is not decomposed protects the soil from the pounding action of rain, without this the soil can become far less permeable. In chaparral vegetated areas, the hydrophobic oils in the succulent leaves can be spread over the soil surface with fire, creating large areas of hydrophobic soil. Other conditions that can lower infiltration rates or block them include dry plant litter that resists re-wetting, or frost. If soil is saturated at the time of an intense freezing period, the soil can become a concrete frost on which almost no infiltration would occur. Over an entire watershed, there are likely to be gaps in the concrete frost or hydrophobic soil where water can infiltrate.

Once water has infiltrated the soil it remains in the soil, percolates down to the ground water table, or becomes part of the subsurface runoff process. The process of infiltration can continue only if there is room available for additional water at the soil surface. The available volume for additional water in the soil depends on the porosity of the soil and the rate at which previously infiltrated water can move away from the surface through the soil. The maximum rate that water can enter a soil in a given condition is the infiltration capacity. If the arrival of the water at the soil surface is less than the infiltration capacity, all of the water will infiltrate. If rainfall intensity at the soil surface occurs at a rate that exceeds the infiltration capacity, pounding begins and is followed by runoff over the ground surface, once depression storage is filled. This runoff is called Horton overland flow. The entire hydrological system of a watershed is sometimes analyzed using hydrology transport models, mathematical models that consider infiltration, runoff and channel flow to predict river flow rates and stream water quality.

The infiltration of incompressible fluid is useful to control salinity of water; contamination of water and agriculture purpose and it also useful in nuclear waste disposable problems. The infiltration model was first developed by Boussinesq in 1903 and is related to the original motivation of Polubarinova Kochina [9], Scheidegger [10], Muskat [2] and Bear J. [1]. Different researchers have discussed this problem with a different point of views. Verma [12] discussed infiltration of incompressible fluid for inclined plain in heterogeneous porous media. Mehta M. N. [6] obtained the solution of singular perturbation technique of one dimensional flow in unsaturated porous media. Mehta and Desai [7] discussed the solution of seepage of ground water in soil by Homotopy perturbation method. Mehta and patel [8] discussed solution of Burger's equation arising into the one dimensional ground water recharge by spreading in porous media. Mehta and Meher [5] discussed the Adomian decomposition method for moisture content in one dimensional fluid flow through unsaturated porous media. Mehta and Yadav [4] discussed the solution of problem arising during vertical ground water recharge by spreading in slightly saturated porous media. Joshi and Mehta [3] apply the Group theoretic approach to the problem of one dimensional fluid flow through unsaturated porous method.

2. Statement of the Problem

Assume that Ground water infiltration take place over large basin area, it enters soil and achieve some water table. The problem is to determine effective height of the water table as measure of the initial storage capacity of the basin.

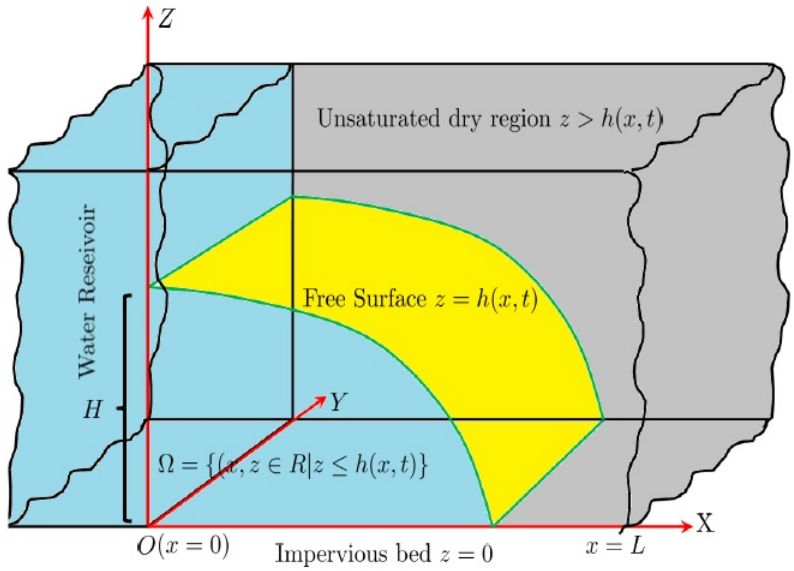

Figure 1: 3-Dimensional view of Infiltration of Ground water in the surrounding soil.

3. Basic Assumptions

Following observations were used to obtain the governing equation for the Infiltration of Ground water.

1. The stratum has height h_m and lies on the top of horizontal impervious bed which we label as $z = 0$.
2. We ignore the transversal variable y and
3. The water mass which infiltrates the soil occupies a region described as

$$\Omega = \left\{(x, z) \in R \middle| z \leq h(x,t)\right\} \tag{1}$$

   and hence we assume that there is no partial saturation. Where,z is called the free boundary function and it determine the height of the free surface and $h(x, t)$ is the maximum height of the free surface.

4. In this situation we arrive at system of three equations in three unknowns; two velocity components and pressure in a variable domain.
   (a) Equation of mass conservation for an incompressible fluid.

(b) Two equations of mass conservation of momentum of the Navier stoke type.

5. The resulting system of equations with initial and boundary conditions is too complicated and in order to simplify the equation, we assume that

   (a) We assume that the flow is horizontal, i.e. $\vec{V} = (u, 0)$ and

   (b) The free boundary function $h(x,t)$ has small gradients.

6. Hence, the simplified view of the 2-dimensional infiltration of Ground water flow is given by,

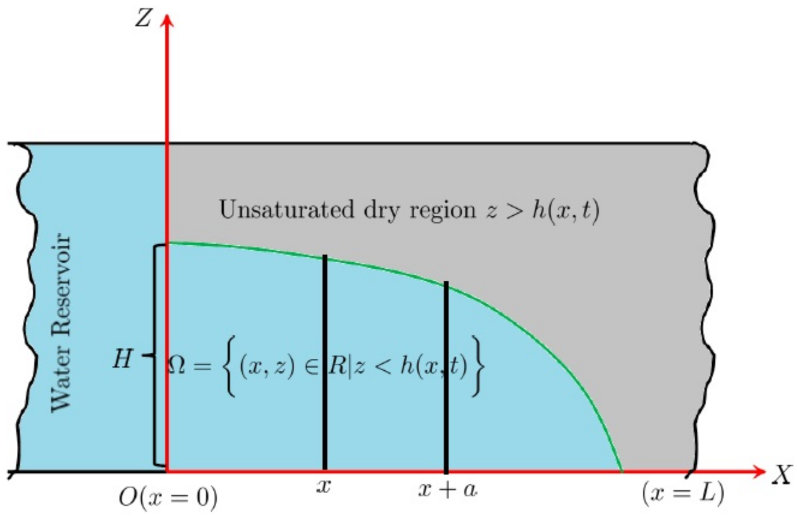

Figure 2: 2-Dimensional view of Infiltration of Ground water in the surrounding soil.

4. Formulation of the Problem

The momentum equation in vertical component(u_z) is given by,

$$\rho\left(\frac{du_z}{dt} + v\nabla u_z\right) = -\frac{\partial p}{\partial z} - \rho g \tag{2}$$

Neglecting the left hand side of above equation(2) then and integrating it with respecting to z, we obtain

$$p + \rho g z = C \tag{3}$$

If we impose the continuity of the pressure across the interface and assuming the constant atmospheric pressure in the air that fills the pore of the dry region

$z > h(x,t)$ and letting $p = 0$ on the surface $z = h(x,t)$ in the equation(3) gives us,

$$\rho g h = C \tag{4}$$

Hence, from equation (2) and (3) we obtain,

$$p = \rho g(h - z) \tag{5}$$

In other words, the pressure is determined by the hydrostatic approximation. Considering the mass conservation law for a section $S = (x, x + a) \times (0, C)$, we get

$$\epsilon \frac{\partial}{\partial t}\left[\int_x^{x+a} \int_0^h \mathrm{d}z\mathrm{d}x\right] = -\int_{\partial S}\left(\vec{V} \cdot n\right)\mathrm{d}l \tag{6}$$

Where, ϵ is the porosity of the medium (i.e fraction of the volume available for flow circulation), $\vec{V}$ denotes the velocity, which is obeys the *Darcy's law* that includes the gravity effect.

$$\vec{V} = -\left(\frac{k}{\mu}\right)\nabla(p + \rho g h) \tag{7}$$

Considering the velocity component of $\vec{V}$ along the lateral surface (i.e $\vec{V} \cdot n = u$), we arrive at

$$u = -\frac{k}{\mu}\left(\frac{\partial p}{\partial x}\right) \tag{8}$$

Hence, from equation(5) and (8), we conclude that

$$u = -\frac{\rho g k}{\mu}\left(\frac{\partial h}{\partial x}\right) \tag{9}$$

Inserting the expression of u in the equation(6), we get

$$\epsilon \frac{\partial h}{\partial t} = \left(\frac{\rho g k}{\mu}\right)\frac{\partial}{\partial x}\left[\int_0^h \frac{\partial}{\partial x} h \cdot \mathrm{d}z\right] \tag{10}$$

The above equation(10) is in the form of non-linear *Boussinesq equation* and which can be rewritten as

$$\frac{\partial h}{\partial t} = \beta \frac{\partial^2}{\partial x^2}\left[h^2\right]; \text{ Where } \beta = \frac{\rho g k}{\epsilon \mu} \tag{11}$$

The above equation(11) gives the water table at any distance x and at any time t. Using the dimensionless variables $T = \left(\frac{2\rho g k}{\epsilon \mu L^2}\right) t$ and $X = \frac{x}{L}$, it can be simplified to

$$\frac{\partial h}{\partial T} = \frac{\partial}{\partial X}\left[f(h)\frac{\partial h}{\partial X}\right]. \text{ Where } f(h) = h. \tag{12}$$

The appropriate initial and boundary conditions for the above problem are given by

$$h(x,0) = h_1 \text{ at } T = 0 \text{ and for any distance } X > 0 \tag{13}$$

$$h(0,T) = 1 \text{ for any time } T > 0 \tag{14}$$

$$h(L,T) = 0 \text{ at } X = L \text{ and for any time } T > 0. \tag{15}$$

$$\frac{\partial h}{\partial X} = -\omega \text{ at time } X = 0. \tag{16}$$

5. Solution of the Problem

Consider the *infinitesimal transformations* of the dependent variable h and two independent variables X and T, where *infinitesimal generator* is given by *twice prolonged operator* Γ as

$$\Gamma = \xi\frac{\partial}{\partial X} + \eta\frac{\partial}{\partial T} + \zeta\frac{\partial}{\partial h} + \zeta_1\frac{\partial^2}{\partial hX} + \zeta_2\frac{\partial^2}{\partial hT} + \zeta_{11}\frac{\partial^3}{\partial hXX} \tag{17}$$

Rewriting the equation(12) as,

$$h_T = F(X,T) = f(h)h_{XX} - f^{'}(h)(h_X)^2 = 0 \tag{18}$$

From equation(17) and equation(18) the invariance condition is given by,

$$\Gamma(h_T)\Big|_{F=0} = \left[\xi\frac{\partial}{\partial X} + \eta\frac{\partial}{\partial T} + \zeta\frac{\partial}{\partial h} + \zeta_1\frac{\partial^2}{\partial hX} + \zeta_2\frac{\partial^2}{\partial hT} + \zeta_{11}\frac{\partial^3}{\partial hXX}\right]h_T\Big|_{F=0} \tag{19}$$

Here *the coordinates of first prolongation* of the *twice prolonged operator* (17) is given by,

$$\zeta_1 = \zeta_X + (\zeta_h - \xi_X)h_X - \eta_X h_T - \xi_h h_X^2 - \eta_h h_X h_T \tag{20}$$

$$\zeta_2 = \zeta_T - \xi_T h_X + (\zeta_h - \eta_T)h_T - \xi_h h_X h_T - \eta_h h_T^2 \tag{21}$$

Similarly *the coordinates of second prolongation* of *the twice prolonged operator* (17) is given by,

$$\begin{aligned}\zeta_{11} = {} & \zeta_{XX} + (2\zeta_{hX} - \xi_{XX})h_X - \eta_{XX}h_T + (\zeta_{hh} - 2\xi_{hX})h_X^2 \\ & - \eta_{hX}h_X h_T - \xi_{hh}h_X^3 - \eta_{hh}h_X^2 h_T + (\zeta_h - 2\xi_X - 3\xi_h h_X - \eta_h h_T)h_{XX} \\ & - 2(\eta_X + \eta_h h_X)h_{XT}.\end{aligned} \tag{22}$$

Substituting the right hand side of equation(18) in the equation(19) and using formula of coordinates of first prolongation(20), (21) and the formula of second prolongation(22) and then equating to zero the coefficients of different powers of remaining derivatives, we obtain the *set of determining equations* for symmetry group of the governing equation(12). The above determining equations are *first order linear partial differential equations* in ξ, η and ζ.

$h_X h_{XX}$	$2f(h)(\eta_{hX} f(h) + \xi_h) + f^{'}(h)\eta_X = 0$
h_{XX}	$\zeta f^{'}(h) - f^2(h)\eta_{XX} - f(h)(2\xi_X - \eta_T) = 0$
$h_X h_{XT}$	$f(h)\eta_h = 0$
h_{XT}	$f(h)\eta_X = 0$
h_X^4	$f^{'}(h)\eta_h + f(h)\eta_{hh} = 0$
h_X^3	$2\left[f^{'}(h)\right]^2 \eta_X + f(h)\xi_{XX} + f^{'}(h)\xi_h + 2f(h)f^{'}(h)\eta_{hX} = 0$
h_X^2	$f(h)\zeta_{hh} + f^{''}(h)\zeta - 2f(h)\xi_{hX} - f^{'}(h)(2\xi_X - \eta_T)$ $+f^{'}(h)\zeta_h - f(h)f^{'}(h)\eta_{hh} = 0$
h_X	$2f(h)\zeta_{hX} + 2f^{'}(h)\zeta_X - f(h)\xi_{XX} + \xi_T = 0$
1	$\zeta_T - f(h)h_{XX} = 0$

Here the first column lists combinations of derivatives and the second column contains the corresponding functional coefficients (up to a constant factor)) identical expression and those obtained by differentiation is omitted. Since $f(h) \neq 0$ from the first equation of the above table, we conclude that

$$\eta = \eta(T) \tag{23}$$

From the equations second equation of the table and (23) we conclude that

$$\xi = \xi(X, T) \tag{24}$$

From the equations (23) and (24), we conclude that

$$\zeta = \frac{f(h)(2\xi_X - \eta_T)}{f^{'}(h)} \tag{25}$$

Taking into account the relations obtained above, we can rewrite the system given by the table in the form

$$[ff^{'}f^{'''} - f(f^{''})^2 + (f^{'})^2 f^{"}](2\xi_X - \eta_T) = 0 \tag{26}$$

$$f[4ff^{''} - 7(f^{'})^2\xi_{XX} - (f^{'})^3]\xi_T = 0 \tag{27}$$

$$2f\xi_{XXX} - \xi_{XT} + \eta_{TT} = 0 \tag{28}$$

In general, for arbitraty $f(h)$, equation (26) implies $(2\xi_X - \eta_T) = 0$ and the equation(27) implies $\xi_T = 0$. From the equation(28), we get $\xi = c_1 + c_2X$ and therefore $\eta = 2c_2T + c_3$. It follows that for arbitrary $f(h)$, solving the above system of linear partial differential equations called *determining equations* which is spanned by *four dimensional Lie-algebra* we obtain the most general *infinite symmetry* of the governing equation(12) as,

$$\Gamma_1 = \frac{\partial}{\partial X}, \Gamma_2 = \frac{\partial}{\partial T}, \Gamma_3 = X\frac{\partial}{\partial X} + 2T\frac{\partial}{\partial T} \text{ and } \Gamma_4 = 3X\frac{\partial}{\partial X} + 2h\frac{\partial}{\partial h} \tag{29}$$

5.1 Invariant solution with respect to operator Γ_3

For an arbitrary function $f(h)$, the *invariant solution* corresponding to operator $\Gamma_3 = 2T\frac{\partial}{\partial T} + X\frac{\partial}{\partial X}$ is given by the following first order linear partial differential equation.

$$\Gamma_3 I = 0 \Rightarrow 2T\frac{\partial I}{\partial T} + X\frac{\partial I}{\partial X} + (0)\frac{\partial I}{\partial h} = 0 \tag{30}$$

The corresponding *characteristic system of ordinary differential equations* from equation(30) are given by,

$$\frac{\mathrm{d}X}{X} = \frac{\mathrm{d}T}{2T} \text{ and } \mathrm{d}h = 0 \tag{31}$$

admits the integrals

$$I_1 = XT^{-1/2} = C_1 \text{ and } I_2 = h = C_2 \tag{32}$$

are invariants of the operator Γ_3. Taking $I_2 = \phi(I_1)$, we get

$$h = \phi(z) \text{ and } z = XT^{-1/2}. \tag{33}$$

Where $\phi(z)$ is a function to be determined in the further analysis. Substituting (31) into the equation(12), we arrive at *the second order nonlinear ordinary differential equation* as,

$$2[f(\phi)\phi_z^{'}]_z^{'} + z\phi_z^{'} = 0 \tag{34}$$

which describes *an invariant (self-similar)* solution. Applying Leibniz's rule evaluating $\mathrm{n^{th}}$ derivative at $z = 0$, we obtain an recurrence relation as,

$$\phi^{n+2}(0) = -\frac{1}{\phi(0)}\Bigg[2\phi^{'}(0)\phi^{n+1}(0)(n+1) + n\phi^{n}(0)[2\phi^{"}(0)+1] \tag{35}$$
$$+ \sum_{k=2}^{n}\binom{n}{k}\Bigg\{2\phi^{k+1}(0)\phi^{n-k+1}(0) + \phi^{k}(0)\phi^{n-k+2}(0)\Bigg\}\Bigg]$$

Applying initial conditions(14) & (16), the abvoe equation(35) we obtain

$$\phi(0) = 1, \phi^{'}(0) = -w, \phi^{"}(0) = -w^2, \tag{36}$$
$$\phi^{'"}(0) = -\left(3w^3 - \frac{w}{2}\right), \phi^{iv}(0) = -\left(\frac{29}{2}w^4 - 3w^3\right)$$

Hence, substituting the value (36), in the Maclaurin's series of $\phi(z)$ we get,

$$\begin{aligned}\phi(z) &= \sum_{k=0}^{\infty} \phi^k(0)\frac{z^k}{k!} \\ &= \phi(0) + z\phi^{'}(0) + \frac{z^2}{2!}\phi^{''}(0) + \phi^{'''}(0)\frac{z^3}{3!} + \frac{z^4}{4!}\phi^{iv}(0) + --\end{aligned}$$

$$\therefore\ \phi(z) = 1 - wz - \frac{w^2z^2}{2!} - \left(3w^3 - \frac{w}{2}\right)\frac{z^3}{3!} - \left(\frac{29}{2}w^4 - 3w^3\right)\frac{z^4}{4!} + -- \tag{37}$$

From the equations(37) & (33), height of the water mound is given by,

$$h(X,T) = 1 - \frac{Xw}{\sqrt{T}} - \frac{w^2X^2}{T2!} - \left(3w^3 - \frac{w}{2}\right)\frac{X^3}{T^{2/3}3!} - \left(\frac{29}{2}w^4 - 3w^2\right)\frac{X^4}{T^2 4!} + -- \tag{38}$$

From the equations(38) & (5), the expression of atmospheric pressure is given by

$$p = \left[\rho g\left(1 - \frac{Xw}{\sqrt{T}} - \frac{w^2X^2}{T2!} - \left(3w^3 - \frac{w}{2}\right)\frac{X^3}{T^{2/3}3!} - \left(\frac{29}{2}w^4 - 3w^2\right)\frac{X^4}{T^2 4!} + --\right) - z\right] \tag{39}$$

From the equation(39) & the equation(8) the expression of velocity is given by,

$$u = -\frac{\rho g k}{\mu}\left[-\frac{w}{\sqrt{T}} - \frac{w^2X}{T} - \left(3w^3 - \frac{w}{2}\right)\frac{X^2}{T^{2/3}2} - \left(\frac{29}{2}w^4 - 3w^2\right)\frac{X^3}{T^2 3!} + --\right] \tag{40}$$

5.2 Convergence study

Equation(37) represents height of the water mound in the form of Maclaurin's series in z and equation(38) represents in original variables X & T. Hence it is sufficient to discuss the convergence of Maclaurin's series in z to discuss convergence of equation(38). From the equation(37), consider

$$u_{k+1} = \frac{\phi^{k+1}(0)z^{k+1}}{(k+1)!} \& u_k = \frac{\phi^k(0)z^k}{(k)!}$$

and hence as per Ratio test

$$\lim_{k\Rightarrow\infty}\frac{|u_{k+1}|}{|u_k|} = \lim_{k\Rightarrow\infty}\frac{\phi^{k+1}(0)}{\phi^k(0)}\left|\frac{z}{k+1}\right| = 0 < 1.$$

5.3 Graphical representation of Height of the water mound and Pressure vs. distance X for a fixed time T obtained with respect to operator Γ_3.

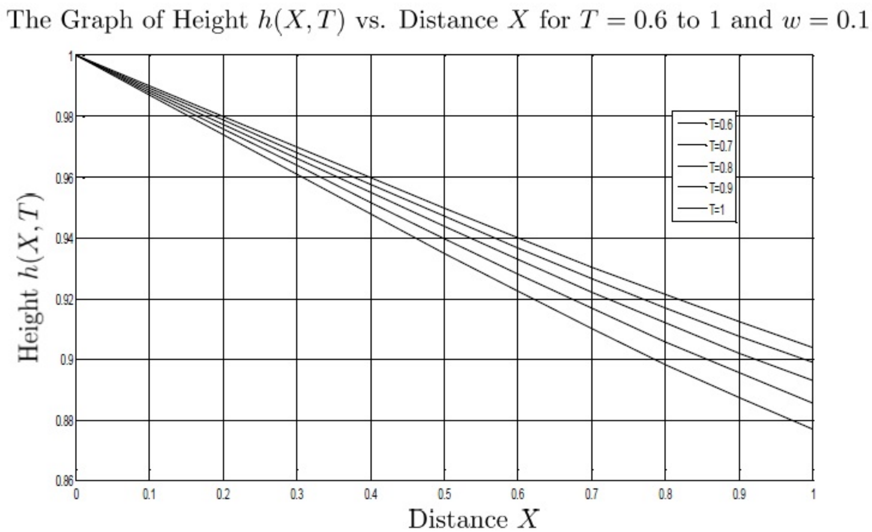

Figure 3: The Graph of Height of water table $h(X,T)$ vs. Distance X for a fixed time $T = 0.6$ to 0.1.

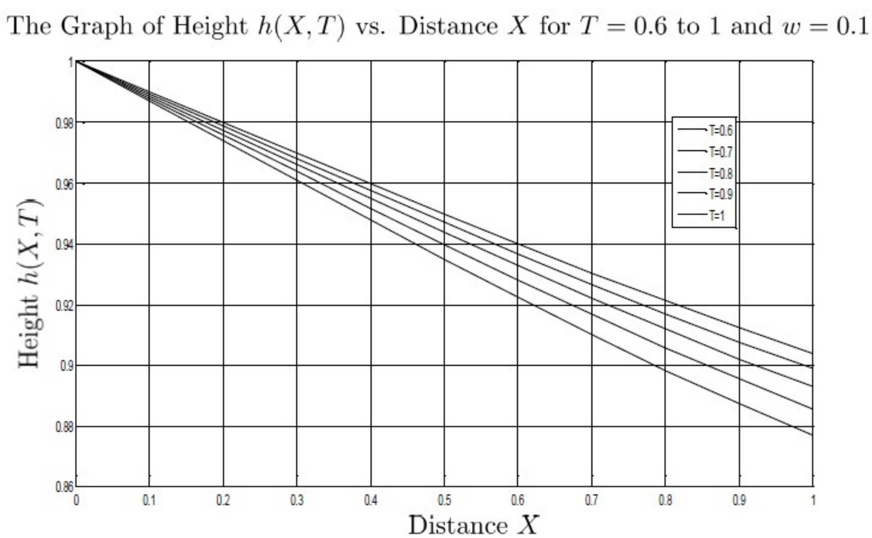

Figure 3: The Graph of Height of water table $h(X,T)$ vs. Distance X for a fixed time $T = 0.6$ to 0.1.

5.4 Invariant solution with respect to operator Γ_4.

The invariant solution corresponding to $f(h) = h$ and with respect to the operator $\Gamma_4 = X\dfrac{\partial}{\partial X} + 2h\dfrac{\partial}{\partial h}$ are describe by the following first order linear partial differential equation.

$$\Gamma_4 I = 0 \Rightarrow \frac{\partial I}{\partial T} + 3X\frac{\partial I}{\partial X} + 2h\frac{\partial I}{\partial h} = 0 \tag{41}$$

The corresponding system of ordinary differential equations from(33) are given by

$$\frac{\mathrm{d}X}{X} = \frac{\mathrm{d}h}{2h} \text{ and } \mathrm{d}T = 0 \tag{42}$$

and hence *the scaling invariant solution* of equation(12) is given by,

$$h(X,T) = X^2\left[A - 6aT\right]^{-1} \tag{43}$$

Where A is an arbitrary constant. Applying initial condition(13) on the above equation(35) the solution is given by

$$h(X,T) = X^2\left[\frac{1}{X^2/h_1 - 6aT}\right] \tag{44}$$

$$h(x,t) = \left(\frac{x}{L}\right)^2\left[\frac{\left(\frac{x}{L}\right)^2}{h_1} - \left(\frac{6a\rho gk}{\epsilon\mu}\right)t\right]^{-1} \tag{45}$$

Hence, the atmospheric pressure is given by

$$p = \rho g\left[\left(\frac{x}{L}\right)^2\left[\frac{\left(\frac{x}{L}\right)^2}{h_1} - \left(\frac{6a\rho gk}{\epsilon\mu}\right)t\right]^{-1}\right] \tag{46}$$

Hence the velocity along the axial direction is given by

$$u = -\frac{\rho gk}{\mu}\left(\frac{\partial h}{\partial x}\right) \tag{47}$$

$$= -\frac{\rho gk}{\mu}\left[2\cdot\left(\frac{x}{L^2}\right)\left[\frac{\left(\frac{x}{L}\right)^2}{h_1} - \left(\frac{6a\rho gk}{\epsilon\mu}\right)t\right]^{-1} - 2\left(\frac{x^3}{L^4 h_1}\right)\left[\frac{\left(\frac{x}{L}\right)^2}{h_1} - \left(\frac{6a\rho gk}{\epsilon\mu}\right)t\right]^{-2}\right]$$

The equation(37) is the solution of governing equation (12) that represents *the height of free surface or water mound of infiltrated water* in unsaturated heterogeneous porous media for any distance for any time which satisfies both boundary and initial conditions.

5.5 Graphical representation of the Height of the water mound and Pressure vs distance X for a fixed time T obtained with respect to operator Γ_4.

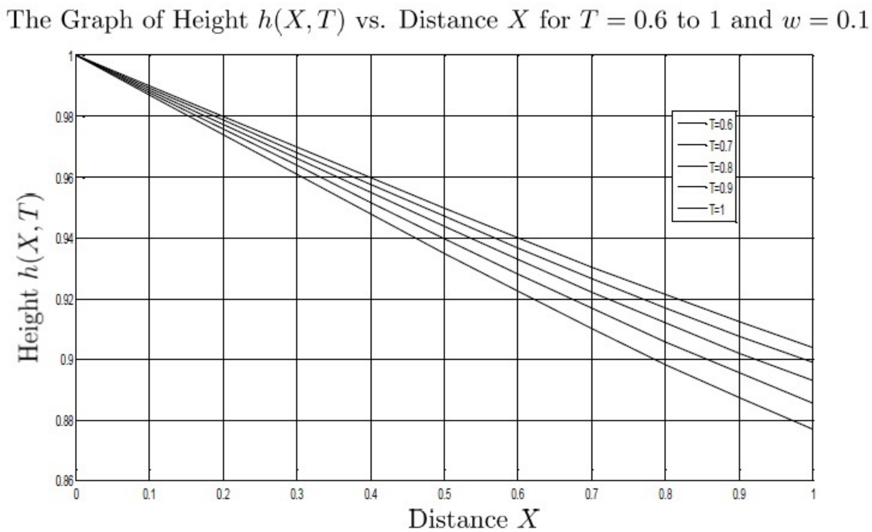

Figure 3: The Graph of Height of water table $h(X,T)$ vs. Distance X for a fixed time $T = 0.6$ to 0.1.

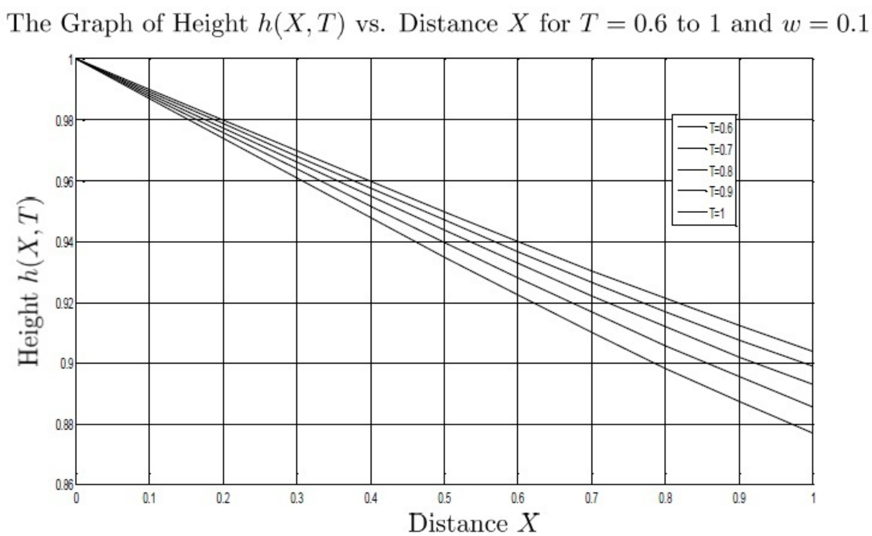

Figure 3: The Graph of Height of water table $h(X,T)$ vs. Distance X for a fixed time $T = 0.6$ to 0.1.

6. Interpretation of the Solution and Discussion

1. Using Lie-group one parameter analysis we obtain Lie-algebra spanned by four operators Γ_1, Γ_2, Γ_3 & Γ_4. The solution corresponding to operators Γ_1 & Γ_2 is of no use as they give solution in only one variable.

2. Hence, we obtain solution corresponding to two operators Γ_3 & Γ_4. The operator Γ_3 leads to similarity solution where as the operator Γ_4 gives the solution in the closed form. Hence, Lie-group analysis method called the generalization of the similarity methods.

3. The equation(37) and the equation(38) gives the height of water mound and pressure as a function of X & T obtained with the help of similarity transformation obtained from the operator Γ_3 and their graphical representation is given by in Figure-1 and Figure-2.

4. Both these graphs in decreasing nature as distance X increase for the fixed time $T = 0.6$ to 1.

5. Equation(45) is the solution of governing equation (12) which represents the height of free surface or water mound of infiltrated water in unsaturated heterogeneous porous media for any distance for any time which satisfied boundary and initial conditions.

6. Figure(3) shows that the height of frees surface or water mound is decreases as X increases for different time $T > 0$ which is experimentally or physically fact, which will decreases to zero as distance increasing to as per figure shown.

7. Figure(3) shows the graph of height of free surface of infiltrated water in homogeneous porous media vs. at distance X for $T = 0.5$ to 0.8.

8. Figure(4) shows the graph of pressure of infiltrated water p vs. distance X for a fixed time T.

7. Acknowledgment

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